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RESEARCH MEMORANDUM

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ANALYSIS OF AN AUTOMATIC CONTROL

TO PREVENT ROLLING DIVERGENCE

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CLASSIFIED DOCUMENT

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NATIONAL ADVISORY COMMITTEE
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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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ANALYSIS OF AN AUTOMATIC CONTROL

TO PREVENT ROLLING DIVERGENCE

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SUMMARY

An analog computer investigation has been made to determine the effectiveness of an automatic control intended to prevent rolling divergence by offsetting the roll coupling terms in the equations of motion. These terms represent yawing and pitching moments applied to the airplane proportional to the products of angular velocities in roll and pitch and in roll and yaw. The effects of operating the rudder and stabilizer to offset or overcompensate for these roll coupling terms have been studied in maneuvers simulating rolls of a fighter airplane. The effects of pilot's control movements of the rudder and stabilizer, and of time lag in the control, have been briefly investigated.

The proposed control was effective in reducing the excursions of an angle of attack and sideslip in the maneuvers studied. Overcompensating for the roll coupling terms was slightly more effective than exactly offsetting them in maneuvers involving unfavorable pilot control coordination, but such overcompensation resulted in a tendency for unstable oscillations when lag was present in the control.

The signals proportional to products of angular velocities required as inputs to the control might be obtained by measuring the angular velocities separately and multiplying them. An alternate method using a gyroscopic instrument to measure these products of angular velocities directly is also described.

INTRODUCTION

Several current fighter airplanes have experienced instability in rapid rolls. This instability has resulted in inadvertent violent maneuvers involving large changes in attitude and excessive structural loads. A description of flight-test results for two such airplanes is presented in reference 1. Provision of inherent aerodynamic stability

sufficient to avoid such difficulties may prove exceedingly difficult, particularly on very high-altitude airplanes, and limitation of the rate of roll and maximum angle of roll appears to be the only method of avoiding difficulty. The degree to which such limitations may be imposed without adversely affecting the tactical usefulness of an airplane has not been definitely established. Nevertheless, the permissible rates of roll may be much lower than those attainable by current fighter airplanes. Furthermore, a roll-rate limiter may entail considerable complication. This complication might better be applied to some device to remove the need for limitation.

The use of automatic controls to reduce the tendency for rolling divergence has been investigated in references 2 and 3. These studies have considered the effect of artificial changes in certain stability derivatives on the rolling divergence, and have shown that increased damping in pitch may be quite effective in reducing the divergent tendency. In the present report, a different type of automatic control is investigated.

The analysis presented in reference 4 has shown that the primary cause of instability in rolling maneuvers is the presence of certain terms in the equations of motion, known as the roll coupling terms, which cause pitching and yawing moments to be applied to the airplane when it is rolling. One method for avoiding rolling instability would therefore appear to be the application of pitching and yawing moments to the airplane through an automatic control to offset these roll coupling terms. A brief analysis has been made on an electronic analog computer to show the effect of such an automatic control. The necessary inputs to this type of automatic control are the products of angular velocities in roll and pitch and in roll and yaw. A gyroscopic instrument for measuring these products of angular velocities is described in appendix A.

The present analysis is considered preliminary in that only one airplane in one flight condition has been studied, and in that no detailed comparison of various types of automatic control has been attempted.

SYMBOLS

B   factor determining gyro sensitivity, H/K

b   wing span

C_{L_α} rate of change of lift coefficient with angle of attack, $\partial C_L/\partial \alpha$

C_{l_p} rate of change of rolling-moment coefficient with rolling-angular-velocity factor, $\dfrac{\partial C_l}{\partial \left(\frac{pb}{2V}\right)}$

C_{l_r} rate of change of rolling-moment coefficient with yawing-angular-velocity factor, $\dfrac{\partial C_l}{\partial \left(\frac{rb}{2V}\right)}$

C_{l_β} rate of change of rolling-moment coefficient with sideslip, $(\partial C_l / \partial \beta)$

$C_{l_{\delta_a}}$ rate of change of rolling-moment coefficient with aileron deflection, $\partial C_l / \partial \delta_a$

$C_{m_{i_t}}$ rate of change of pitching-moment coefficient with stabilizer incidence, $\partial C_m / \partial i_t$

C_{m_q} rate of change of pitching-moment coefficient with pitching-angular-velocity factor, $\dfrac{\partial C_m}{\partial \left(\frac{q\bar{c}}{2V}\right)}$

C_{m_α} rate of change of pitching-moment coefficient with angle of attack, $(\partial C_m / \partial \alpha)$

$C_{m_{\dot\alpha}}$ rate of change of pitching-moment coefficient with rate of change of angle-of-attack factor, $\dfrac{\partial C_m}{\partial \left(\frac{\dot\alpha \bar{c}}{2V}\right)}$

C_{n_β} rate of change of yawing-moment coefficient with sideslip, $\partial C_n / \partial \beta$

C_{n_p} rate of change of yawing-moment coefficient with rolling-angular-velocity factor, $\dfrac{\partial C_n}{\partial \left(\frac{pb}{2V}\right)}$

C_{n_r} rate of change of yawing-moment coefficient with yawing-angular-velocity factor, $\dfrac{\partial C_n}{\partial \left(\frac{rb}{2V}\right)}$

$C_{n_{\delta_r}}$ rate of change of yawing-moment coefficient with rudder deflection, $\partial C_n / \partial \delta_r$

C_{y_β} rate of change of side-force coefficient with sideslip, $\partial C_y / \partial \beta$

$\bar{c}$ mean aerodynamic chord

g acceleration of gravity

H angular momentum of gyro

H_e angular momentum of engine

h altitude

$I_{x_0}, I_{y_0}, I_{z_0}$ moments of inertia about airplane principal axes

i_t stabilizer incidence

K spring-restoring gradient of gyro

L_0 aerodynamic rolling moment about airplane principal X-axis

l_3, m_3, n_3 direction cosines describing orientation of airplane axis system (see ref. 3)

M_0 aerodynamic pitching moment about airplane principal Y-axis

m mass

N_0 aerodynamic yawing moment about airplane principal Z-axis

p rolling angular velocity; differential operator

q pitching angular velocity; dynamic pressure

r yawing angular velocity

S wing area

V true airspeed

W airplane weight

Y force along airplane Y-axis

Z force along airplane Z-axis

α angle of attack of principal longitudinal axis

α_0 angle of attack of principal longitudinal axis in straight flight

β angle of sideslip

δ_a total aileron deflection

δ_g deflection of gyro gimbal

ϕ angle of roll

τ time lag of automatic control

ω_1 angular velocity about an axis perpendicular to undisplaced gyro spin axis and perpendicular to gimbal axis

ω_2 angular velocity about an axis parallel to undisplaced gyro spin axis

Dot over symbol denotes differentiation with respect to time.

All stability derivatives are given with respect to principal axes.

ANALYSIS

The equations of motion of an airplane with respect to principal axes, assuming airspeed constant, are as follows:

$$mV(\dot{\alpha} - q + p\beta) - mgn_3 = Z$$

$$mV(\dot{\beta} + r - p\alpha) - mgm_3 = Y$$

$$I_{x_0}\dot{p} - \left(I_{y_0} - I_{z_0}\right)qr = L_0$$

$$I_{y_0}\dot{q} - \left(I_{z_0} - I_{x_0}\right)pr + H_e r = M_0$$

$$I_{z_0}\dot{r} - \left(I_{x_0} - I_{y_0}\right)pq - H_e q = N_0$$

$$\dot{l}_3 = m_3 r - n_3 q$$

$$\dot{m}_3 = n_3 p - l_3 r$$

$$\dot{n}_3 = l_3 q - m_3 p$$

In the present analysis it is proposed to examine the effects of offsetting the underlined terms which represent the pitching and yawing moments applied to the airplane through roll coupling. A block diagram illustrating the proposed automatic control is shown in figure 1. These roll coupling terms could be offset, for example, by sensing rolling, yawing and pitching velocities of the airplane by means of instruments alined to record these quantities about the principal axes. The products pq and pr would then be obtained, multiplied by appropriate constants and applied as inputs to servomechanisms to operate the rudder and stabilizer, respectively.

In order to provide improved damping of pitching and yawing oscillations of an airplane in normal flight, automatic controls are frequently provided in the form of yaw dampers and pitch dampers. The present automatic control could utilize the same servos to operate the controls and could also utilize the same devices to sense yawing and pitching velocity as are conventionally used for the pitch and yaw dampers. A more complicated circuit or mechanism for combining these inputs in the desired manner with a rolling velocity signal would, of course, be required. An alternate arrangement in which the product terms pq and pr are sensed directly by suitable gyroscopic instruments could also be employed. A gyroscopic instrument for performing this function is described in the appendix A.

An additional source of disturbance which may prove undesirable in some airplanes is the gyroscopic moment of the engine. It would appear logical to combine the provisions for yaw and pitch damping, offsetting roll coupling terms, and offsetting engine gyroscopic moments in the same automatic control mechanism. A block diagram of an automatic control incorporating these provisions is shown in figure 1(b). This control is not discussed in detail, inasmuch as only the simpler type of control to offset the roll coupling terms is analysed in the present report. It may be mentioned, however, that the canceler shown in the yaw damper channel is a device to reduce steady-state signals to zero. Such a device has generally been found desirable in yaw dampers in order to prevent the control from opposing the pilot in a steady turn.

In order to make a preliminary analysis of the effectiveness of an automatic control which offsets the roll coupling terms, transient solutions were obtained on a Reeves electronic analog computer for the motions of an airplane with and without the roll coupling terms included in the equations. In addition, some solutions were obtained with the sign of the roll coupling terms reversed to correspond to a condition in which an automatic control overcompensates for these terms by providing twice as much control as necessary to exactly counteract them. The equations of motion as set up on the analog computer are given in appendix B and the assumed values of the stability derivatives are given in table I.

In the analysis in which the roll coupling terms were omitted or reversed no account was taken of lag in the servomechanisms used to

operate the control surfaces. In order to determine the effect of lag
in this type of control, some additional solutions were obtained in which
the dynamics of the control and servomechanisms were approximated by a
first order lag. In other words, the moments supplied by the elevator
and rudder were multiplied by the expression $\dfrac{1}{1 + \tau p}$. The time constant,
τ, which represents the time required for the control to reach 0.63 times
its final deflection following a step input of the control signal, was
varied from 0 to 0.1 second in these runs.

RESULTS AND DISCUSSION

The results of the analysis are presented as time histories of the
computed airplane motion in figures 2 to 8. Only left rolls are presented
because the motion in left rolls is somewhat more violent than in right
rolls under the conditions investigated. This difference occurs because
of the engine gyroscopic effect. Figure 2 illustrates the motion of the
airplane in a 360° roll made with 20° total aileron at a Mach number of
0.7 and an altitude of 32,000 feet. This flight condition is used through-
out the report. Curves are presented for three cases: basic airplane,
roll coupling terms omitted from the equations, and signs of the roll
coupling terms reversed. Figure 3 presents a similar sequence of runs
in a roll using 20° total aileron which was held until about a 720° roll
was completed.

Figure 2 presents a maneuver for the basic airplane similar to that
which was described in reference 1 as having been encountered inadvertently
in flight on a fighter airplane. As the airplane rolls to the left, the
angle of attack goes from positive to negative values. Just before
recovery from the maneuver an angle of attack of about $-3\frac{1}{2}°$ is reached
which would produce an acceleration of approximately -0.8g. In addition,
the sideslip reaches a maximum value of -15.5°. The divergent tendency
present with the basic airplane is eliminated when the roll coupling terms
are removed. In this maneuver, the maximum change in sideslip is only
about 4° and the angle of attack remains positive. Overcompensating for
the inertia terms results in somewhat greater magnitudes of sideslip and
angle-of-attack change in this maneuver than when these effects are exactly
compensated.

The initial variations of angle of attack and sideslip, which appar-
ently result from a conversion of the angle of attack of the principal
axis into sideslip as a result of the tendency of the airplane to rotate
about the principal axis, are not much changed by the automatic control.
The first peak in sideslip of the controlled airplane is close to the
initial angle of attack of 4.8°. In cases of rolling maneuvers in which

the initial angle of attack of the principal axis is large enough that an equivalent sideslip would produce significant vertical tail loads, therefore, some other means for reducing these loads would probably be required.

The control deflections applied by the automatic control are not presented in figure 2 because the procedure of simply omitting the roll coupling terms or reversing their sign did not lend itself to easily recording the required control deflections. The control deflections for a similar maneuver are presented, however, in a subsequent figure showing the effects of lag in the control.

Figure 3 shows results for the roll carried to 720°. In this case, the airplane reaches still greater values of angle of attack and sideslip. The angle of attack and sideslip start to return to zero while the ailerons are deflected, but a very large sideslip angle is reached after the ailerons are returned to neutral. The records were not plotted beyond this point because some component of the analog computer was overloaded. Removing the roll coupling terms or overcompensating for them are again seen to have very beneficial effects.

The effect of compensating for and then overcompensating for the inertia terms in a severely uncoordinated maneuver during which 20° right rudder was applied in conjunction with 20° left aileron is shown for a 360° roll and 720° roll in figures 4 and 5 respectively. While such a maneuver is unlikely to be encountered in practice, it serves to show the effect of this type of automatic control under very adverse conditions. A progressive reduction in the excursions of angle of attack was achieved by compensating and then by overcompensating for the inertia terms. The sideslip angle variations during the maneuver were increased somewhat by compensating for the roll coupling terms. A steady sideslip angle of 10.5° would be expected from the rudder deflection alone, however. Overcompensating for the roll coupling terms reduced the sideslip variations. In effect, overcompensating for the roll coupling terms increases the directional and longitudinal stability of the airplane while it is rolling, thereby reducing the response to applied moments.

It is difficult to obtain or present a sufficient number of examples to show the effectiveness of a particular type of automatic control for preventing rolling divergence on a single airplane in all possible situations because of the large number of combinations of control movements which the pilot may apply and because of the large variations in static longitudinal and directional stability and other important variables such as initial inclination of the principal axis with respect to the flight path which occur on a given airplane with variation in Mach number and angle of attack. One feature of the rolling divergence problem which has been revealed both by flight tests and analog computer studies is the fact that maneuvers encountered are relatively unpredictable from the pilot's standpoint.

Thus, even though a properly coordinated maneuver may be made without causing excessive motions in angle of attack and sideslip, slightly different coordination of the controls may result in a very violent maneuver. This condition exists because the airplane while rolling is unstable. A rapid maneuver may be made without encountering large divergence provided the initial disturbances, which may come from aileron yawing moments, initial inclination of the principal axis with respect to the flight path, and rudder and stabilizer movements, are small or tend to compensate each other. However, the use of control coordination which applies a disturbing moment during the maneuver may rapidly result in a large divergence because of the instability. Flight records showing a large effect of small stabilizer movements on the sideslip encountered in rolls are presented in reference 5.

An analysis similar to that of reference 4 but with the roll coupling terms omitted shows that the rolling airplane will then be stable under all conditions in which it is stable when not rolling. The proposed control would be expected to be very effective in avoiding divergence due to small inadvertent control movements because it makes the airplane stable while it is rolling.

The example airplane and flight conditions used in the time histories presented herein are not very suitable for illustrating the possible large effects of inadvertent pilot control movements. This characteristic occurs because, in the cases chosen, a relatively large disturbing moment is applied to the airplane because of the initial inclination of the principal axis with respect to the flight path. If this disturbing moment were removed, a rapid roll might easily be made without encountering large changes in sideslip or angle of attack. However, small movements of the stabilizer might result in motions similar to those which have been presented. Such control movements which aggravate the motion may result from the natural reactions of the pilot. This effect is illustrated in figure 6 which shows a 360° roll made with 20° left aileron. These conditions are the same as those of figure 2. In this run, the pilot was assumed to apply 2° stabilizer deflection in the direction to produce positive pitching moment when the roll angle had reached 180°. Examination of the curves of figure 6 for the basic airplane shows that a rapidly decreasing angle of attack occurs in conjunction with positive pitching velocity. The pilot, who is more likely to be influenced by a decrease in normal acceleration during the roll than by the positive pitching velocity, may apply stabilizer control in the nose-up direction in an attempt to maintain positive values of normal acceleration. If such a control movement is applied when the pitching velocity is positive, the motion is likely to be aggravated because the product of rolling velocity and pitching velocity acts in the roll coupling term to produce a yawing moment on the airplane. The sideslip reached in this maneuver was -26.5° compared to -15.5° without the stabilizer movement as shown in figure 2. Again the automatic control was effective in preventing large changes in angle of attack and sideslip.

The effect of time lag in the automatic control is shown in fig-
ures 7 and 8. Figure 7 shows the effect of increasing the time constant
τ from 0 to 0.1 second when the control with zero lag just compensates
for the roll coupling terms. Figure 8 shows a similar series of runs
for the case in which the control moments are twice those required to
offset the roll coupling terms. The figures show that the effectiveness
of the control which just compensates for the roll coupling terms is only
slightly reduced by lag of the amount studied. In the case in which the
roll coupling terms are overcompensated, however, lag in the control is
much more detrimental. In this case, the airplane exhibits a short-period
oscillation in angle of attack and sideslip while it is rolling. These
oscillations rapidly diverge when lag is present in the control.

The stabilizer and rudder deflections applied by the automatic control
are shown in figures 7 and 8. The maneuvers for the case of zero time lag
correspond closely to those shown in figure 2. The stabilizer deflection
is quite small, whereas the rudder deflection reaches about 25° in the
case of the control which just offsets the roll coupling terms. The
assumed rudder effectiveness is quite low, however. The value of $C_{n_{\delta_r}}$
is only about one-third that provided on many airplanes. The effect of
increased rudder effectiveness would be to cause a corresponding reduction
in the deflection required.

The rudder deflections shown in the unstable cases of figure 8 are
excessively large and would exceed the available range. The effect of
limiting the rudder deflection was not studied, however, because these
cases are of little practical interest.

Inasmuch as the desired effect of the automatic control to offset
the roll coupling terms is to apply pitching and yawing moments to the
airplane in proportion to the corresponding products of angular velocities,
gain changes would be required in the control as a function of flight
condition to provide control deflections corresponding to this require-
ment. At subsonic speeds, the gains should vary inversely with dynamic
pressure whereas in the transonic and supersonic range this variation
should be modified to take into account the variation of control effec-
tiveness with Mach number. The gain change required for yaw and pitch
dampers in order to maintain oscillations with a given percent of critical
damping would be inversely as the square root of dynamic pressure for
subsonic conditions. In general, therefore, if the control is combined
with conventional yaw dampers, the gain change as a function of flight
condition would differ from that used on the pitch and yaw dampers.
Provision for this feature is shown in figure 1(b) by including separate
blocks for gain changers for the various control inputs before they are
combined and fed to the servo. Some deviation from the ideal programming
of gains might, of course, be permissible in an actual case. However,
the preceding analysis has shown that if the gain of the automatic control

to offset the roll coupling terms is allowed to increase very much beyond the required value, the control will become critical to lags in its operation.

While no analog computer studies have been made to compare the effectivness of pitch and yaw dampers with that of the proposed automatic control, some remarks on the relative effectiveness of the two types of control may be made on the basis of the theory of reference 4. This theory indicates that the use of pitch and yaw dampers broadens the stability region, so that a greater difference between the natural frequencies in pitch and yaw is permissible without encountering instability while rolling. On the other hand, the proposed control makes the airplane stable while rolling for all combinations of natural frequencies in pitch and yaw, provided only that the airplane has static stability in pitch and yaw when it is not rolling. Inasmuch as large variations in C_{m_α} and C_{n_β} inevitably occur due to Mach number effects, the natural frequencies in pitch and yaw are often widely different in certain flight conditions. For these regimes, the proposed control might be more effective than pitch and yaw dampers. On the other hand, the pitch and yaw dampers contribute to the damping of oscillations set up during transient control motions, whereas the proposed control does not add damping. Since such oscillations are often responsible for large excursions in angles of attack and sideslip, it is likely that artificially increased damping would be desirable in any case. For this reason, a combination of the proposed control with pitch and yaw dampers would appear to offer the greatest benefit in avoiding undesirable or dangerous motion in rolls.

In the present report no detailed consideration has been given to the actual design of an automatic control of the type proposed. If the angular velocities in pitch, roll and yaw are measured separately and converted to electrical signals by suitable transducers, various methods are available for obtaining the desired product terms. For example, a servo whose output is proportional to one of the quantities may be used to operate a potentiometer whose excitation is provided by a voltage proportional to the other quantity. The output of the potentiometer will then be proportional to the product of the two quantities. With some types of pickoffs the output of the pickoff on one gyro might be used to excite the pickoff on the other gyro, thereby obtaining an output proportional to the product of the angular velocities in a simple manner. Various other methods of multiplication of electrical quantities have been described in the literature (ref. 6). Another method which has possible application to the automatic control described herein is the use of a gyroscopic instrument which allows the determination of the products of angular velocities by addition of quantities proportional to gyro gimbal deflections rather than by multiplication of these quantities. This instrument is not necessarily recommended in preference to more conventional methods, but it is described because it might prove to be desirable in certain cases. The theory of this instrument is presented in appendix A.

CONCLUDING REMARKS

A preliminary analysis has been made of an automatic control to prevent rolling divergence. This control operates to offset the roll coupling terms in the equations of motion, which represent pitching and yawing moments proportional to the products of angular velocities pr and pq, respectively. In the cases investigated, which are typical for a high-speed fighter airplane, the control is found to be effective in reducing the excursions in angle of attack and in sideslip during rapid rolls.

Inasmuch as conventional pitch and yaw dampers may also be effective in reducing the tendency for rolling divergence, the need for the addition of a control of the type discussed when pitch and yaw dampers are incorporated should be investigated. In addition, further study of the effect of the proposed control when used in combination with pitch and yaw dampers would be desirable. A control of the type proposed, however, is expected to be more generally effective for preventing rolling divergence than pitch and yaw dampers because it makes the airplane stable while rolling under all conditions in which it is stable when not rolling; that is, whenever the static longitudinal and directional stabilities are positive.

Langley Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
        Langley Field, Va., December 22, 1955.

APPENDIX A

A GYROSCOPIC INSTRUMENT FOR MEASURING THE PRODUCT OF TWO

ANGULAR VELOCITIES ABOUT MUTUALLY PERPENDICULAR AXES

The steady-state deflection of the gimbal of a conventional spring-restrained rate gyroscope is given by the formula

$$\delta_g = \frac{H(\omega_1 \cos \delta_g + \omega_2 \sin \delta_g)}{K} \tag{1}$$

where ω_1 is the angular velocity intended to be measured and ω_2 is the angular velocity about an axis perpendicular to the axis about which ω_1 is measured and parallel to the undisturbed gyro spin axis. Normally the gyro restraining spring is made stiff so that the gimbal deflection δ_g is very small and the unwanted term $\omega_2 \sin \delta_g$ is only a small correction. In the present application, however, this term is used to measure the products of angular velocities $\omega_1 \omega_2$. To see how this measurement may be accomplished, assume that δ_g is a small angle so that $\sin \delta_g \approx \delta_g$ and $\cos \delta_g \approx 1$. Let $B = H/K$. Then formula (1) becomes

$$\delta_g \approx B(\omega_1 + \omega_2 \delta_g) \tag{2}$$

or

$$\delta_g \approx \frac{B\omega_1}{1 - B\omega_2} = \frac{B\omega_1(1 + B\omega_2)}{1 - B^2\omega_2^2} = \frac{B\omega_1 + B^2\omega_1\omega_2}{1 - B^2\omega_2^2}$$

In the arrangement under consideration two identical gyros with opposite spin direction are employed, as shown in figure 9. The sum of the gimbal deflections is then:

$$\delta_{g_1} + \delta_{g_2} \approx \frac{2B^2\omega_1\omega_2}{1 - B^2\omega_2^2} \tag{3}$$

The difference between the gimbal deflections is:

$$\delta_{g_1} - \delta_{g_2} \approx \frac{2B\omega_1}{1 - B^2\omega_2^2} \tag{4}$$

Thus, the sum of the gimbal deflections is proportional to the product of the angular velocities $\omega_1\omega_2$, and the difference between the gimbal deflections is a measure of the angular velocity ω_1 without any first-order correction for ω_2.

In order for the proposed instrument to be practical, it is necessary that the sum and difference of the gimbal deflections be sufficiently large to measure accurately and that the natural frequency of the instrument be sufficiently high. Also, as shown by formulas (3) and (4), it is necessary that the product $\beta^2\omega_2'^2$ be small compared to one if the calibration of the instrument is to remain reasonably consistent. An additional question that arises in connection with the application of this type of instrument to an automatic control to prevent rolling divergence is the best orientation of the gyroscopes to sense the products of angular velocities pq and pr. This choice may be affected by the fact that in practice the magnitude of p is considerably larger than that of q and r. Another question which may determine the choice of gyro orientation is whether it is desired to obtain measurements of p, or of q and r, as byproducts of the system.

In order to study whether the instrument is practical and to determine the best gyro orientation, calculations have been made for an instrument using gyroscopes having the following characteristics:

Rotor diameter, in. 2
Rotor moment of inertia (spin axis), slug-ft^2 . . . . 0.000052
Rotor speed, rpm 12,000
Rotor angular momentum, slug-ft^2/sec 0.065
Moment of inertia (gimbal axis), slug-ft^2 . . . . 0.00007

Reasonable values for maximum angular velocities of a fighter airplane are as follows:

p, radians/sec 3
q, radians/sec 1
r, radians/sec 1

First consider the case in which the gyroscopes are oriented with their spin axes parallel to the axis about which the smaller angular velocity occurs. Figure 10 shows the sum of the gimbal deflections $\delta_{g_1} + \delta_{g_2}$ which is available for measuring the values of pq or pr for values of undamped natural frequency of the gyroscope of 10, 15, and 20 cycles per second. These curves are based on exact calculations in which no small-angle approximation was applied to the sine and cosine terms of formula (1).

The initial slope of the calibration curves as given by formula (3) is also plotted in figure 10. Formula (3) is seen to be inadequate to predict accurately the initial slope in the case of the lower values of natural frequency where the rolling velocity is large, although this formula gives the correct initial slope when the rolling velocity is small and the values of q and r approach 0. The reason for this discrepancy is that the large rolling velocity produces an initial deflection of the gimbals which is large enough to violate the small-angle approximation used in the derivation of formula (2). A more accurate estimate of the initial slope may be made by the following procedure. Let the sine and cosine terms be approximated by the first two terms of their series expansions as follows:

$$\cos \delta_g = 1 - \frac{\delta_g{}^2}{2}$$

$$\sin \delta_g = \delta_g - \frac{\delta_g{}^3}{6}$$

If these values are substituted in formula (1), the formula may be rearranged to give

$$\delta_g(1 - B\omega_2) = B\omega_1 \left(1 - \frac{\delta_g{}^2}{2}\right) - B\omega_2 \frac{\delta_g{}^3}{6} \tag{5}$$

Now assume that δ_g is small and that $\omega_2 \ll \omega_1$, so that for the higher order terms on the right-hand side of formula (5), it is reasonable to substitute the relation from formula (2) that $\delta_g = B\omega_1$. Then the formula may be solved for δ_g and the sum of gimbal deflections of two oppositely rotating gyros may be added to give the formula:

$$\delta_{g_1} + \delta_{g_2} = 2B^2\omega_1\omega_2 \frac{1 - \frac{2}{3} B^2\omega_1{}^2}{1 - B^2\omega_2{}^2} \qquad (\omega_2 \ll \omega_1) \tag{6}$$

The initial slope of the calibration curve given by formula (6) is also plotted in figure 10. This formula may be seen to give a better approximation of the initial slope.

From the curves of figure 10 it is apparent that the sum of the gimbal deflections for the values of natural frequency in the range from 10 to 20 cycles per second are sufficiently large to use for accurate measurement purposes. At the value of natural frequency of 10 cycles per second the initial slope of the calibration curve is considerably affected by rolling velocity and by yawing velocity and the curve becomes nonlinear with increasing values of the product pq or pr. Actually, a value of

natural frequency of 10 cycles per second is too low for application to
an automatic control of the type being considered. The curves for this
frequency are included mainly to show more clearly the nature of the
errors in the instrument at large gimbal deflections. These errors are
acceptably small at values of natural frequency of 15 or 20 cycles per
second. For the gyro orientation under consideration the difference
between the gimbal deflections gives as a byproduct the rolling veloc-
ity p. The difference between the gimbal deflections is of the order of
10 to 20 times the sum of the deflections used for measuring the products
of angular velocities pq and pr.

Now consider the case in which the gyroscopes are oriented with their
spin axes parallel to the axis about which larger angular velocity, p,
occurs. This arrangement has the advantage of making available values
of q and r for possible use as inputs to pitch and yaw dampers, or
for an automatic control such as that shown in figure 1(b). In this
manner all the input quantities for this type of automatic control could
be obtained from four gyroscopes (two measuring q and pq and two meas-
uring r and pr). The sum of the gimbal deflections available with
this arrangement as computed by the exact method are given in figure 11(a).
It may be seen that the gimbal deflections available for measuring pq
or pr are about the same as in the previous case at the higher values
of natural frequency. The curves again show some nonlinearity at the
lower value of natural frequency and their initial slope is a function
of the rolling velocity. The effect of rolling velocity is small, how-
ever, at values of natural frequency of 15 or 20 cycles per second.
Because the operation of an automatic control sensitive to pq or pr
does not appear to be critically dependent on the gain constant employed,
this arrangement would probably be acceptable, at least for natural fre-
quencies greater than about 15 cycles per second.

The difference between the gimbal deflections, which furnishes a
measurement of q or r, is plotted in figure 11(b). An effect of p
on the slope of the calibration curves is again apparent. This effect
is much less than would exist with a single gyro oriented with its spin
axis parallel to the roll axis, however. For example, in the case of
the gyro with a natural frequency of 20 cycles per second, a value of
p of 3 radians per second would change the slope of the calibration
curve by 18 percent, whereas with the twin gyros the slope would be
changed by only 3 percent.

APPENDIX B

EQUATIONS OF MOTION AND VALUES OF STABILITY

DERIVATIVES USED IN ANALYSIS

The equations of motion and stability derivatives are given with respect to principal axes.

$$I_x \dot{p} - (I_y - I_z)qr = \delta_a C_{l_{\delta_a}} qSb + pC_{l_p} \frac{qSb^2}{2V} + rC_{l_r} \frac{qSb^2}{2V} + \beta C_{l_\beta}(\alpha)qSb$$

$$I_y \dot{q} - (I_z - I_x)pr + H_e r = i_t C_{m_{i_t}} qS\bar{c} + qC_{m_q} \frac{qS\bar{c}^2}{2V} + \dot{\alpha} C_{m_{\dot{\alpha}}} \frac{qS\bar{c}^2}{2V} + \Delta\alpha C_{m_\alpha} qS\bar{c}$$

$$I_z \dot{r} - (I_x - I_y)pq - H_e q = \delta_r C_{n_{\delta_r}} qSb + rC_{n_r} \frac{qSb^2}{2V} + pC_{n_p} \frac{qSb^2}{2V} + \beta C_{n_\beta} qSb$$

$$mV(\dot{\beta} + r - \alpha p) - mgm_3 = \beta C_{y_\beta} qS$$

$$mV(\dot{\alpha} - q + \beta p) - mgn_3 = -\alpha C_{L_\alpha} qS$$

$$\alpha = \alpha_o + \Delta\alpha$$

$$\dot{l}_3 = m_3 r - n_3 q$$

$$\dot{m}_3 = n_3 p - l_3 r$$

$$\dot{n}_3 = l_3 q - m_3 p$$

1. NACA High-Speed Flight Station: Flight Experience With Two High-Speed Airplanes Having Violent Lateral-Longitudinal Coupling in Aileron Rolls. NACA RM H55A13, 1955.

2. Gates, Ordway B., Jr., Weil, Joseph, and Woodling, C. H.: Effect of Automatic Stabilization on the Sideslip and Angle-of-Attack Disturbances in Rolling Maneuvers. NACA RM L55E25b, 1955.

3. Weil, Joseph, and Day, Richard E.: An Analog Study of the Relative Importance of Various Factors Affecting Roll Coupling. NACA RM H56A06, 1956.

4. Phillips, William H.: Effect of Steady Rolling on Longitudinal and and Directional Stability. NACA TN 1627, 1948.

5. Finch, Thomas W., Peele, James R., and Day, Richard E.: Flight Investigation of the Effect of Vertical-Tail Size on the Rolling Behavior of a Swept-Wing Airplane Having Lateral-Longitudinal Coupling. NACA RM H55L28a, 1956.

6. Davis, Sidney A.: 31 Ways to Multiply. Control Eng. vol. 1, no. 3, Nov. 1954.

TABLE I

VALUES OF THE STABILITY DERIVATIVES AND OTHER

CONSTANTS REQUIRED IN THE ANALYSIS

I_{x_O}, slug-ft^2 10,976

I_{y_O}, slug-ft^2 57,100

I_{z_O}, slug-ft^2 64,975

q, lb/sq ft 197

S, sq ft 376

b, ft 36.6

$\bar{c}$, ft 11.32

W, lb 23,900

m, slugs 742

V, ft/sec 691

h, ft 32,000

M 0.7

g, ft/sec^2 . . . . 32.2

H_e, $\dfrac{\text{slug-ft}^2}{\text{sec}}$ 17,554

α_O, deg 4.8

$C_{l_{\delta_e}}$ per radian . . . . −0.0528

C_{l_p} per radian . . . . −0.255

C_{l_r} per radian 0.042

$C_{l_\beta}(\alpha)$ . . . .
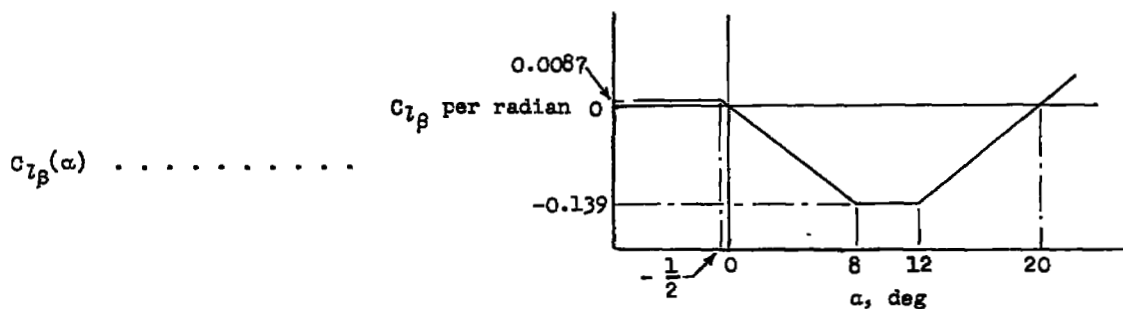
$C_{m_{i_t}}$ per radian . . . . −1.0

C_{m_q} per radian . . . . −3.5

$C_{m_{\dot\alpha}}$ per radian . . . . −1.5

C_{m_α} per radian . . . . −0.36

$C_{n_{\delta_r}}$ per radian . . . . −0.03

C_{n_r} per radian . . . . −0.095

C_{n_p} 0

C_{n_β} per radian 0.057

C_{y_β} per radian . . . . −0.28

C_{L_α} per radian 3.85
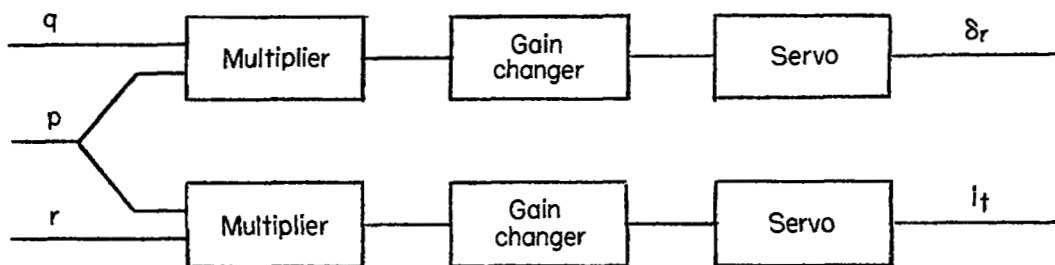
(a) Control to offset roll coupling terms.
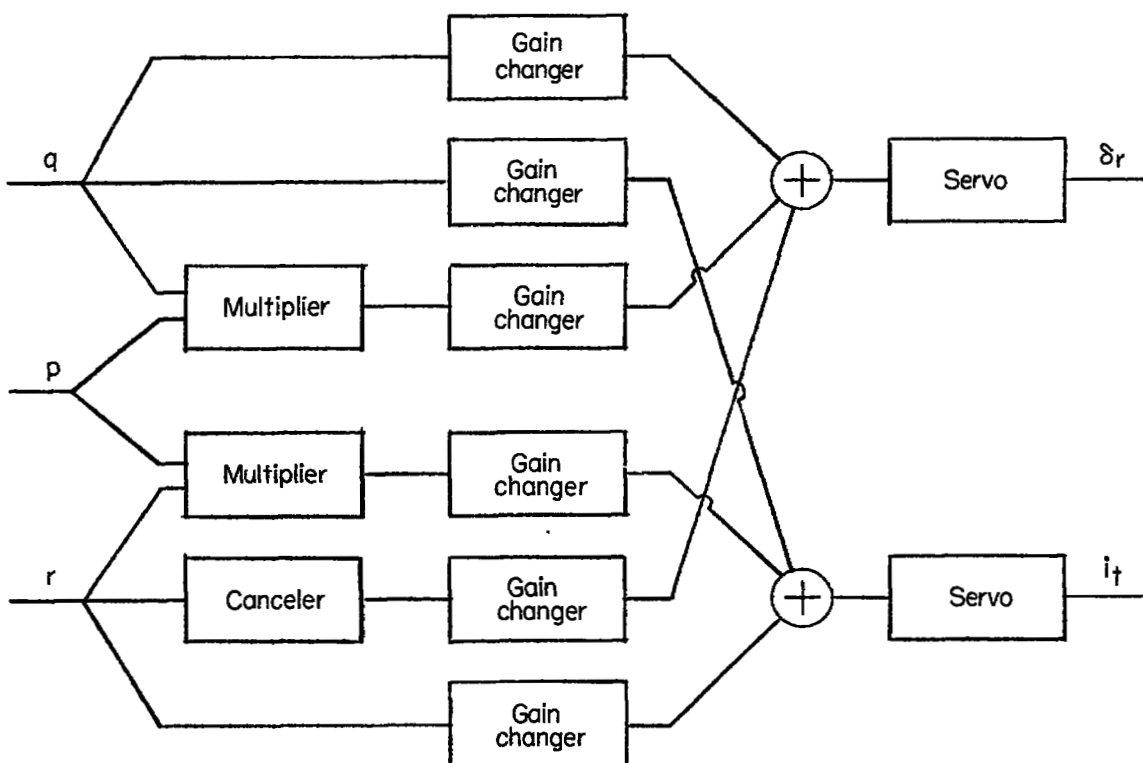
(b) Control to provide yaw and pitch damping, offset roll coupling
terms, and offset engine gyroscopic moments.

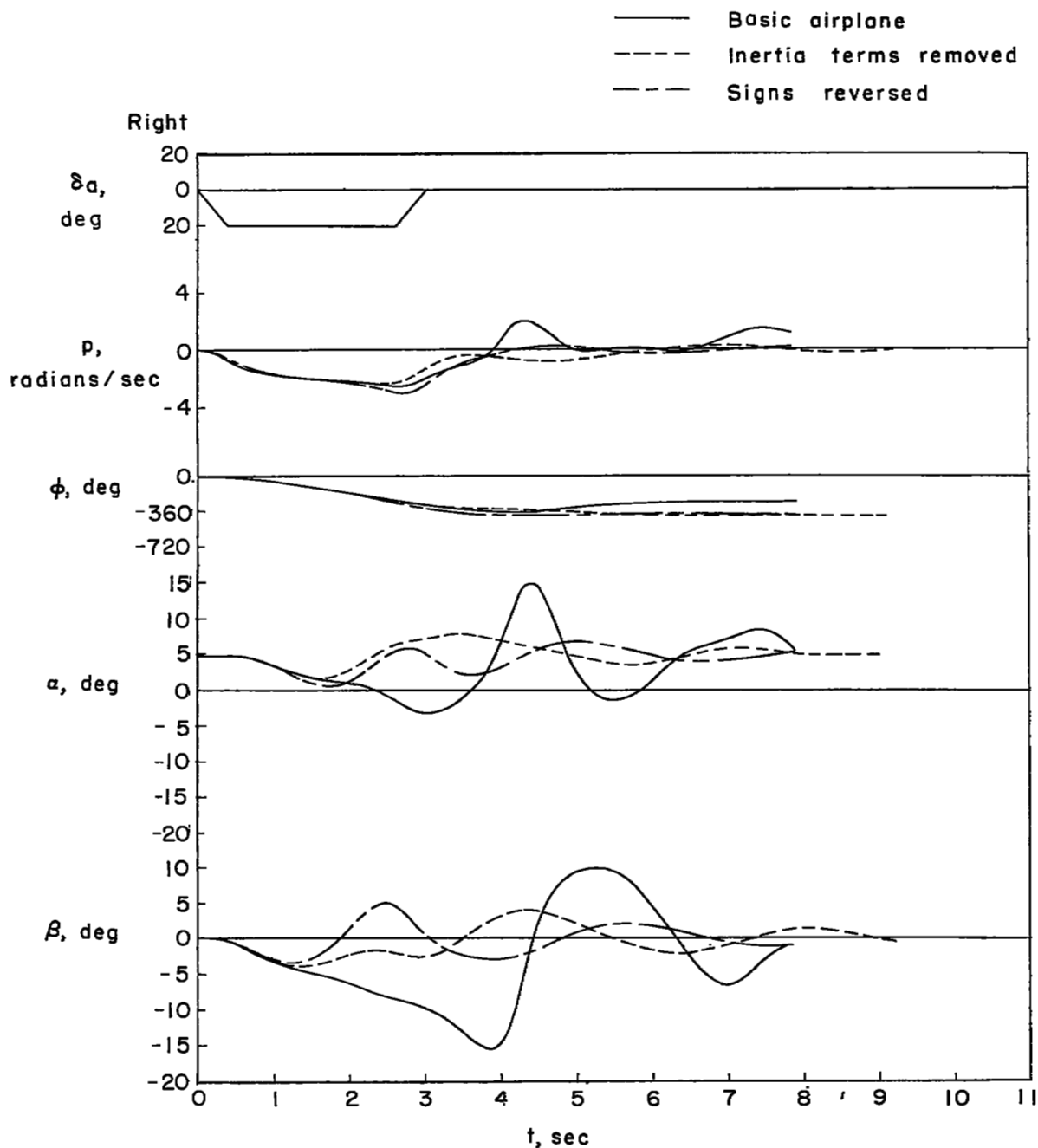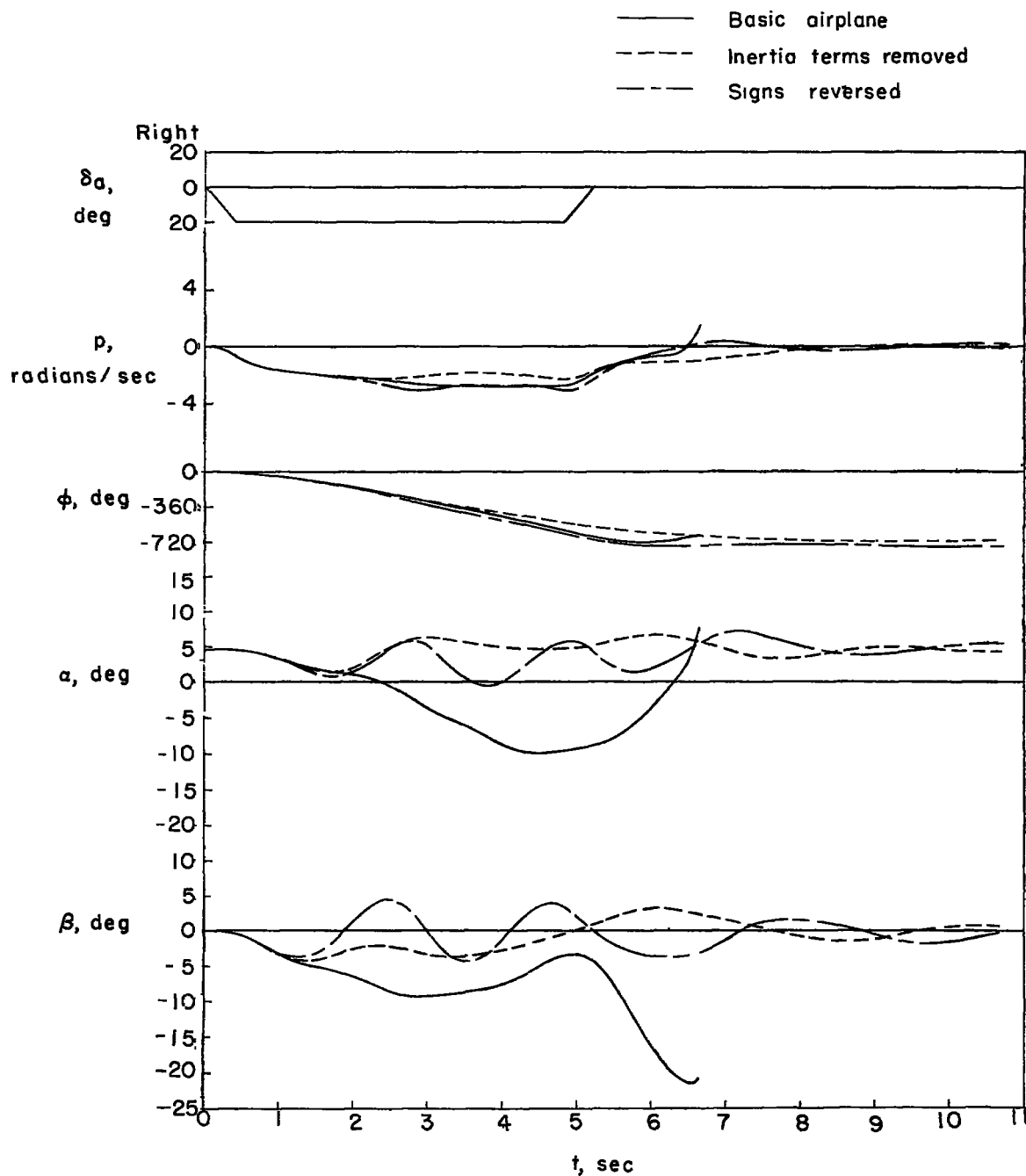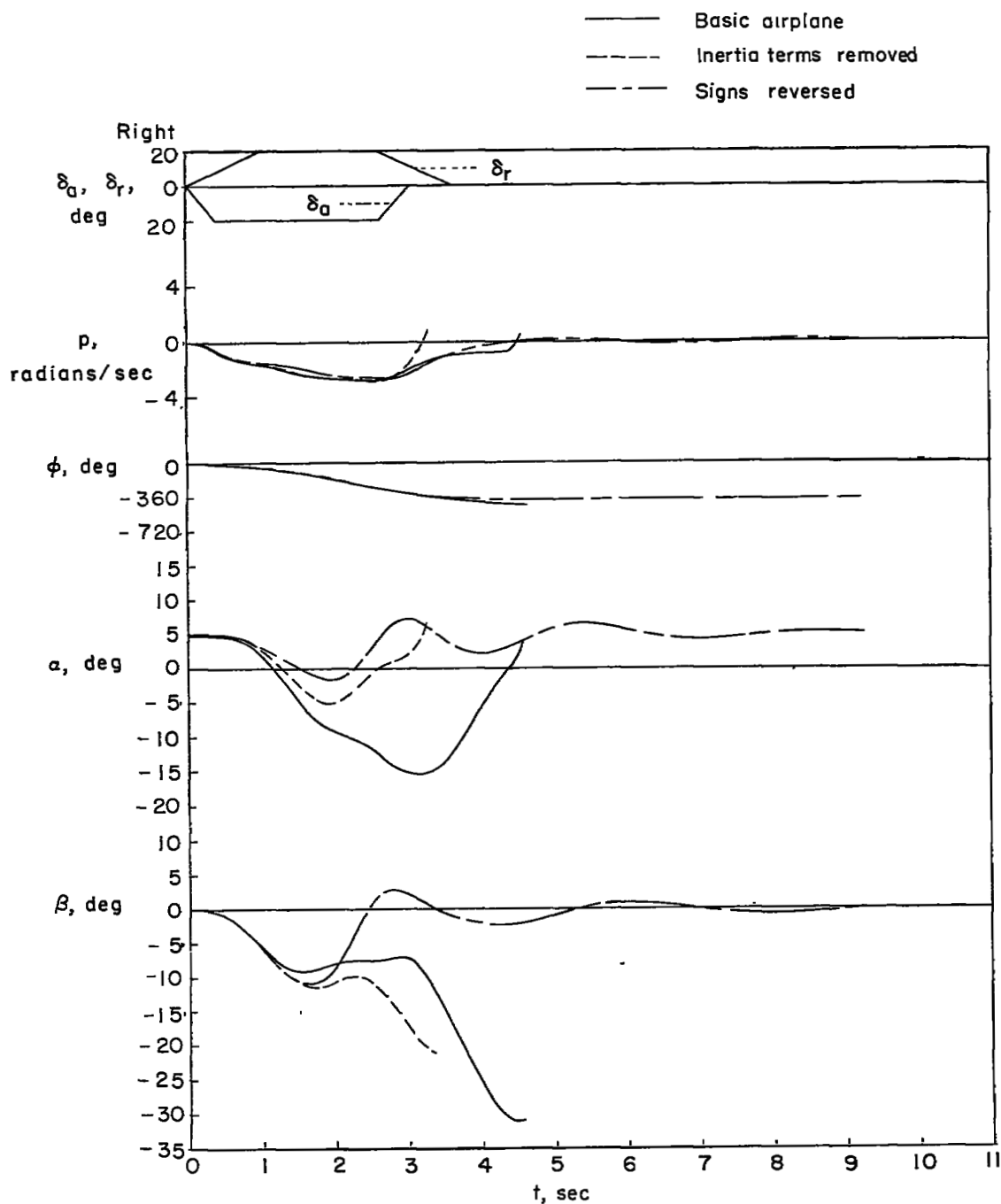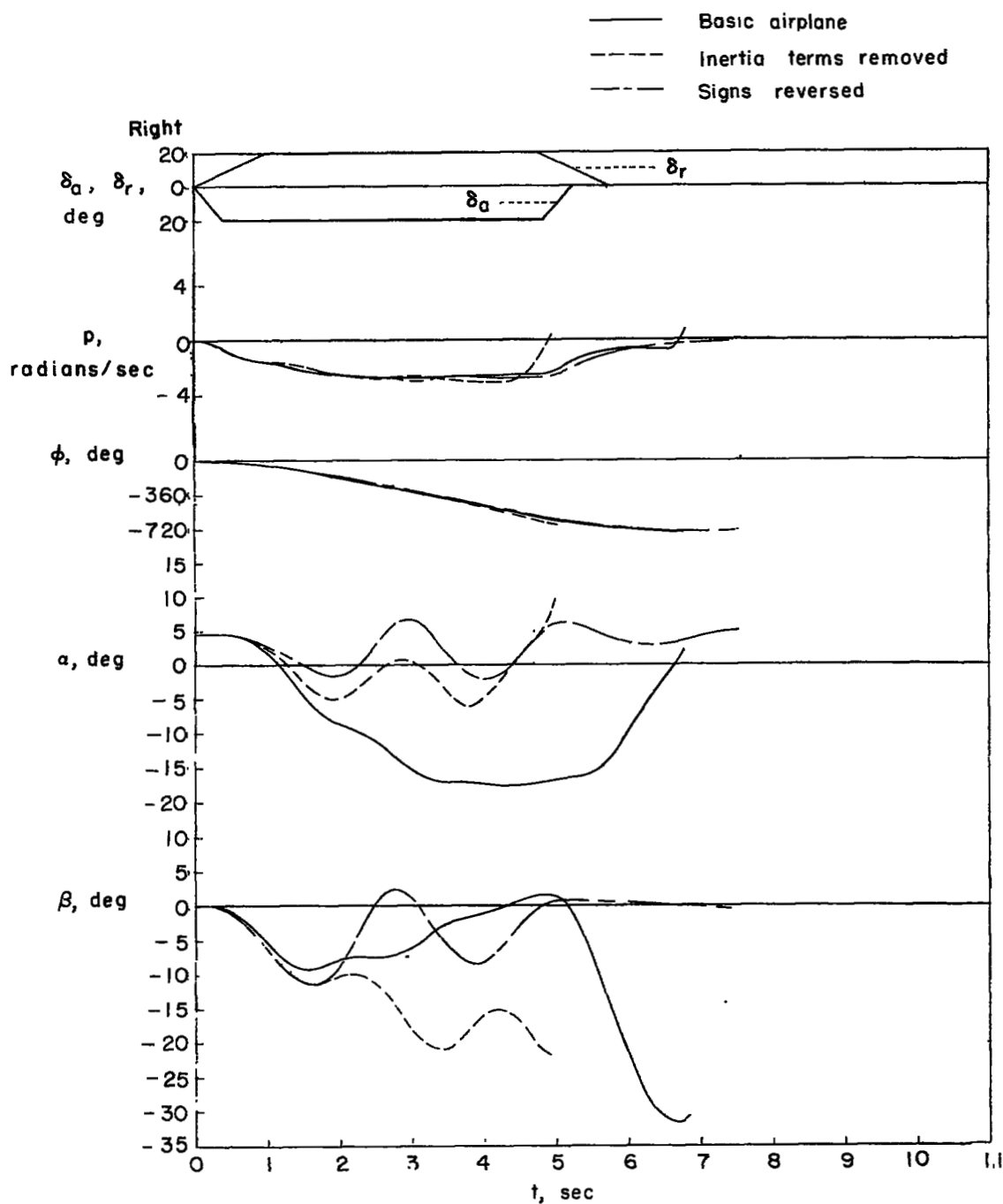Figure 1.- Block diagrams of proposed automatic control mechanisms.

Figure 2.- Time histories of control motion and computed airplane response in 360° rolls made with 20° left total aileron at a Mach number of 0.7 and an altitude of 32,000 feet. Curves shown for basic airplane, roll coupling terms omitted from equations, and signs of roll coupling terms reversed.
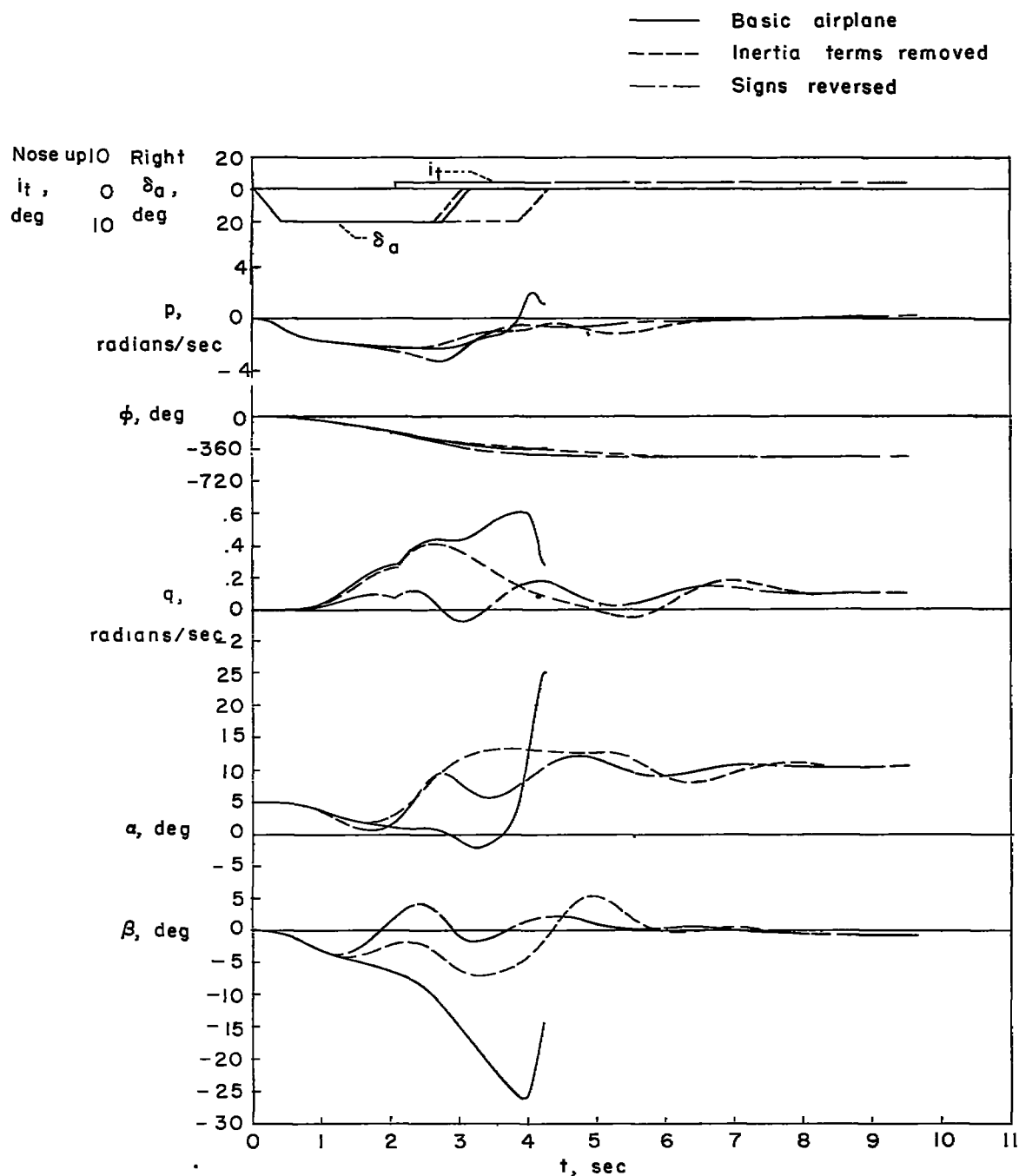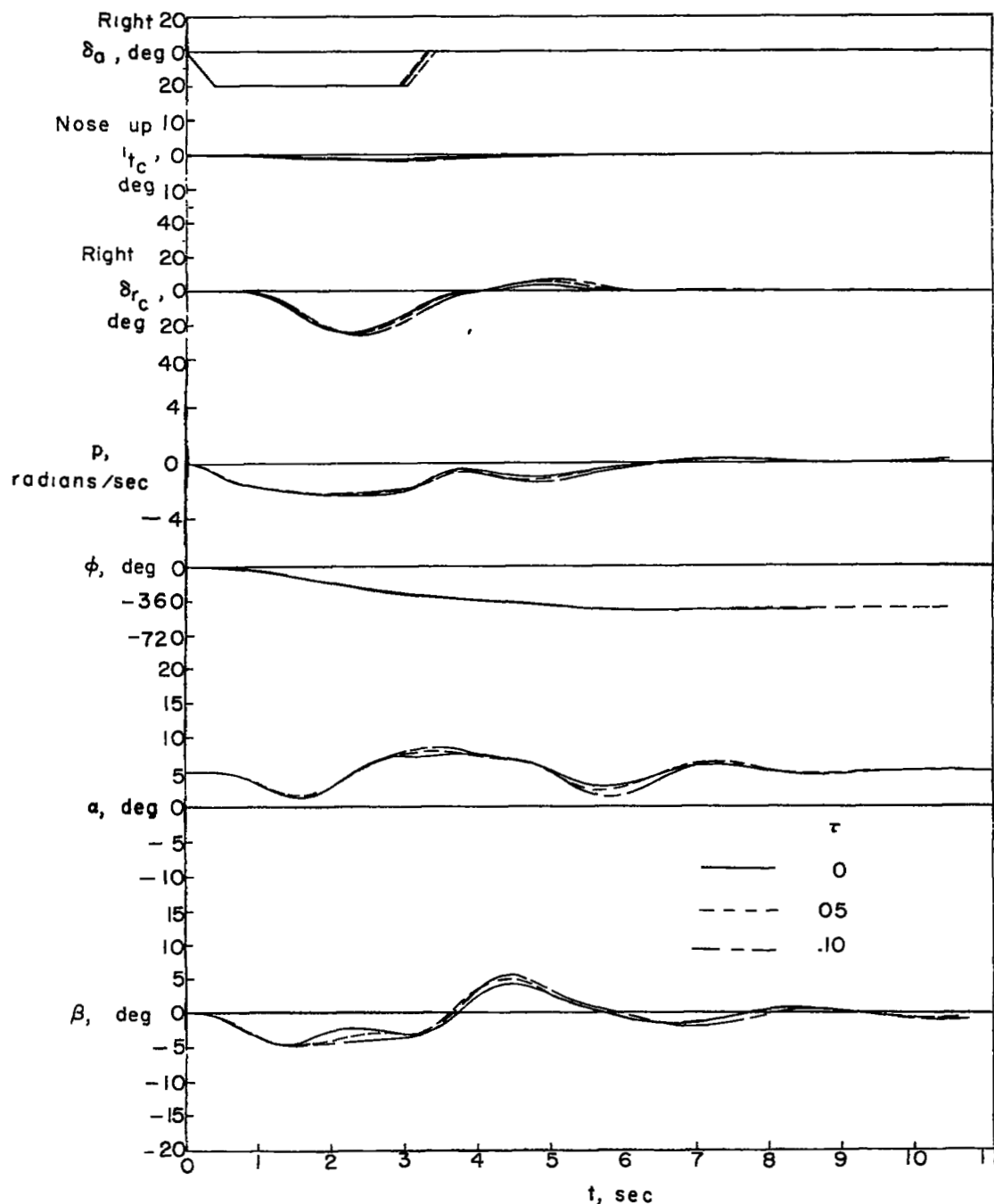
Figure 3.- Time histories of control motion and computed airplane response
in 720° rolls made with 20° left total aileron. Curves shown for basic
airplane, roll coupling terms omitted from equations, and signs of roll
coupling terms reversed.

Figure 4.- Time histories of control motion and computed airplane response in 360° rolls made with 20° left total aileron and 20° right rudder. Curves shown for basic airplane, roll coupling terms omitted from equations, and signs of roll coupling terms reversed.
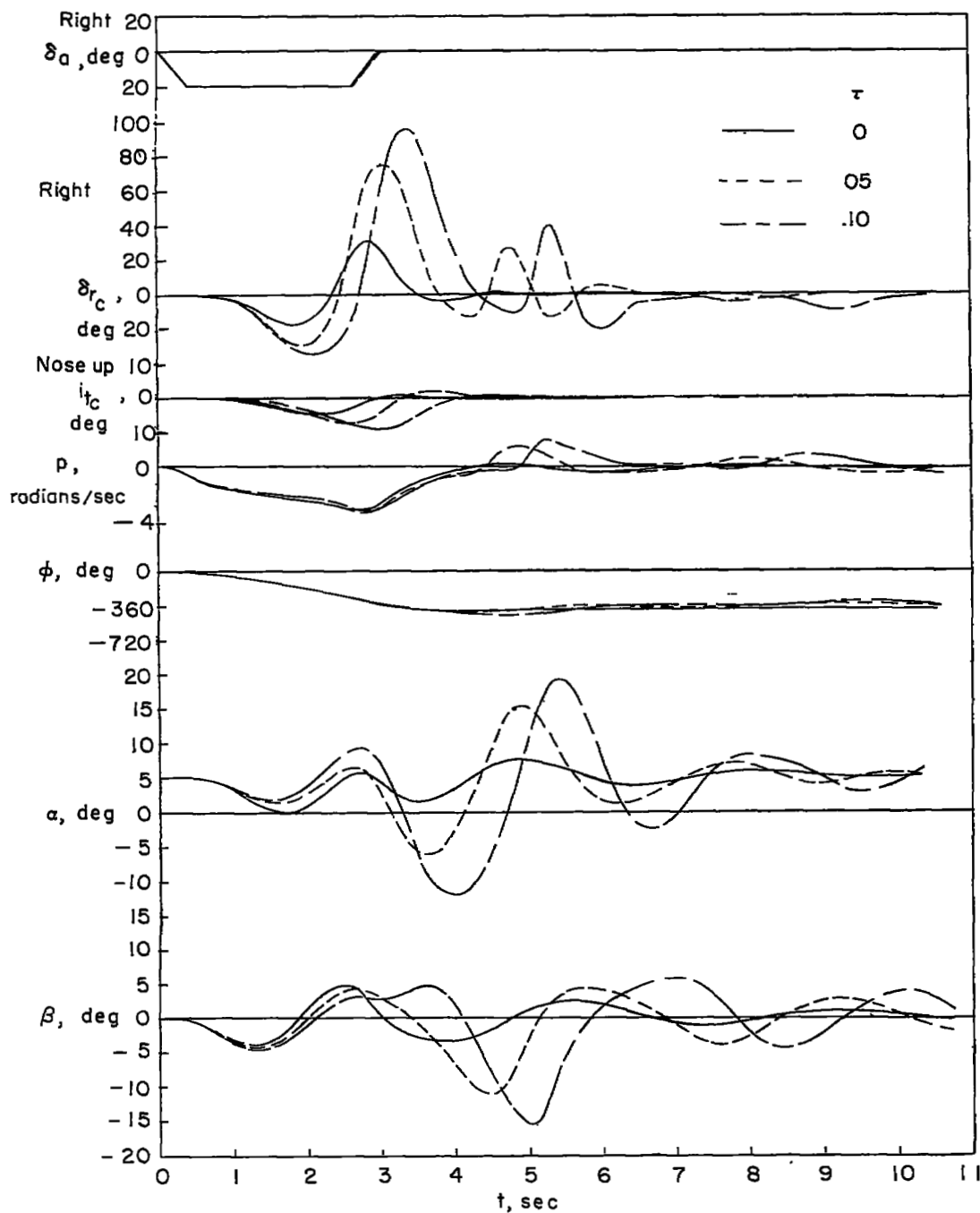
Figure 5.- Time histories of control motion and computed airplane response in 720° rolls made with 20° left total aileron and 20° right rudder. Curves shown for basic airplane, roll coupling terms omitted from equations, and signs of roll coupling terms reversed.

Figure 6.- Time histories of control motion and computed airplane response in 360° rolls made with 20° left total aileron. Stabilizer moved 2° in direction to produce positive pitching moment when roll angle reached 180°. Curves shown for basic airplane, roll coupling terms omitted from equations, and signs of roll coupling terms reversed.

Figure 7.- Time histories of control motion and computed airplane response in 360° rolls made with 20° left total aileron. Curves show effect of varying time lag in the control when the control with zero lag just offsets the roll coupling terms.

Figure 8.- Time histories of control motion and computed airplane response in 360° rolls made with 20° left total aileron. Curves show effect of varying time lag in control when the control with zero lag provides twice the control moments required to just offset the roll coupling terms.
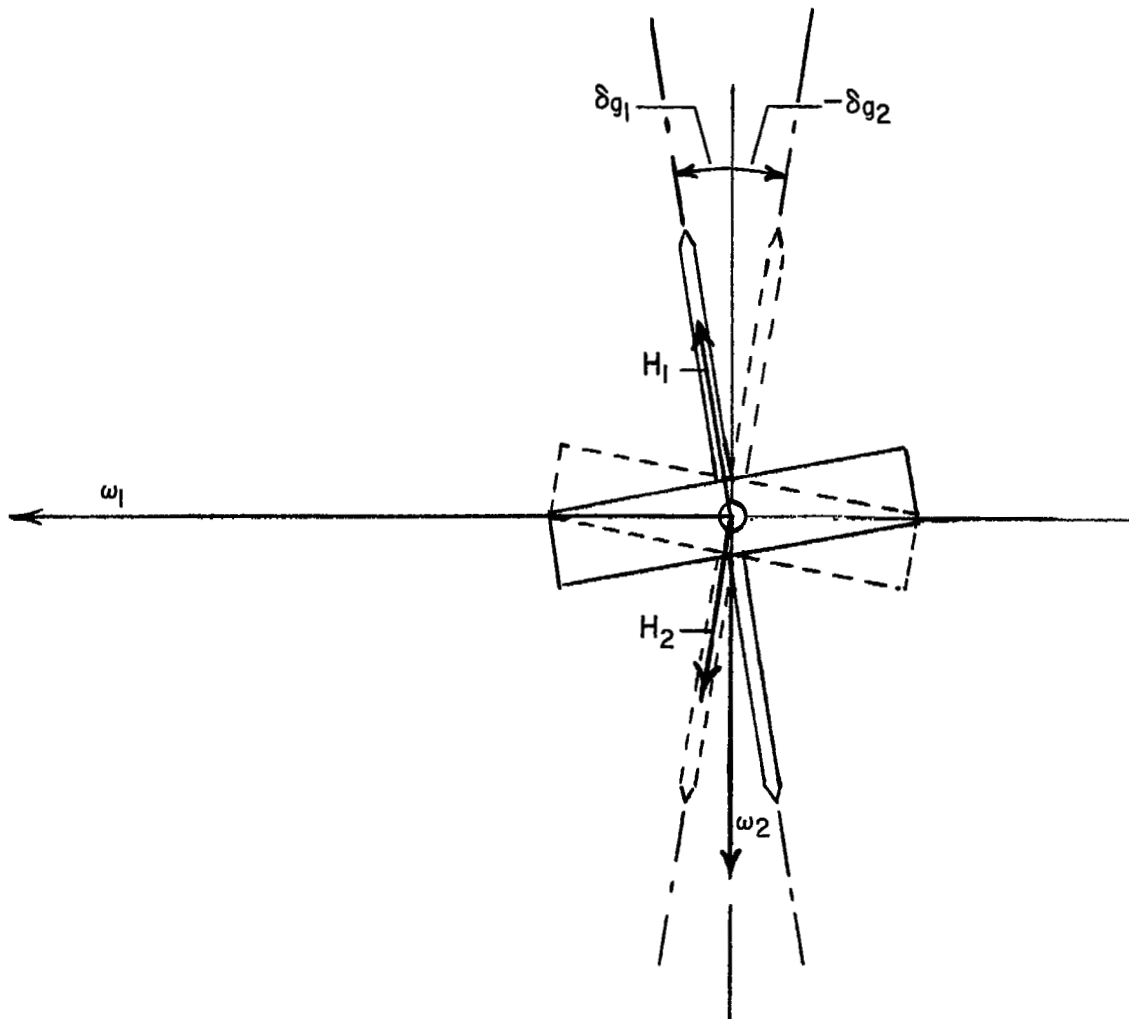
Figure 9.- Arrangement of gyros to measure product of angular veloci-
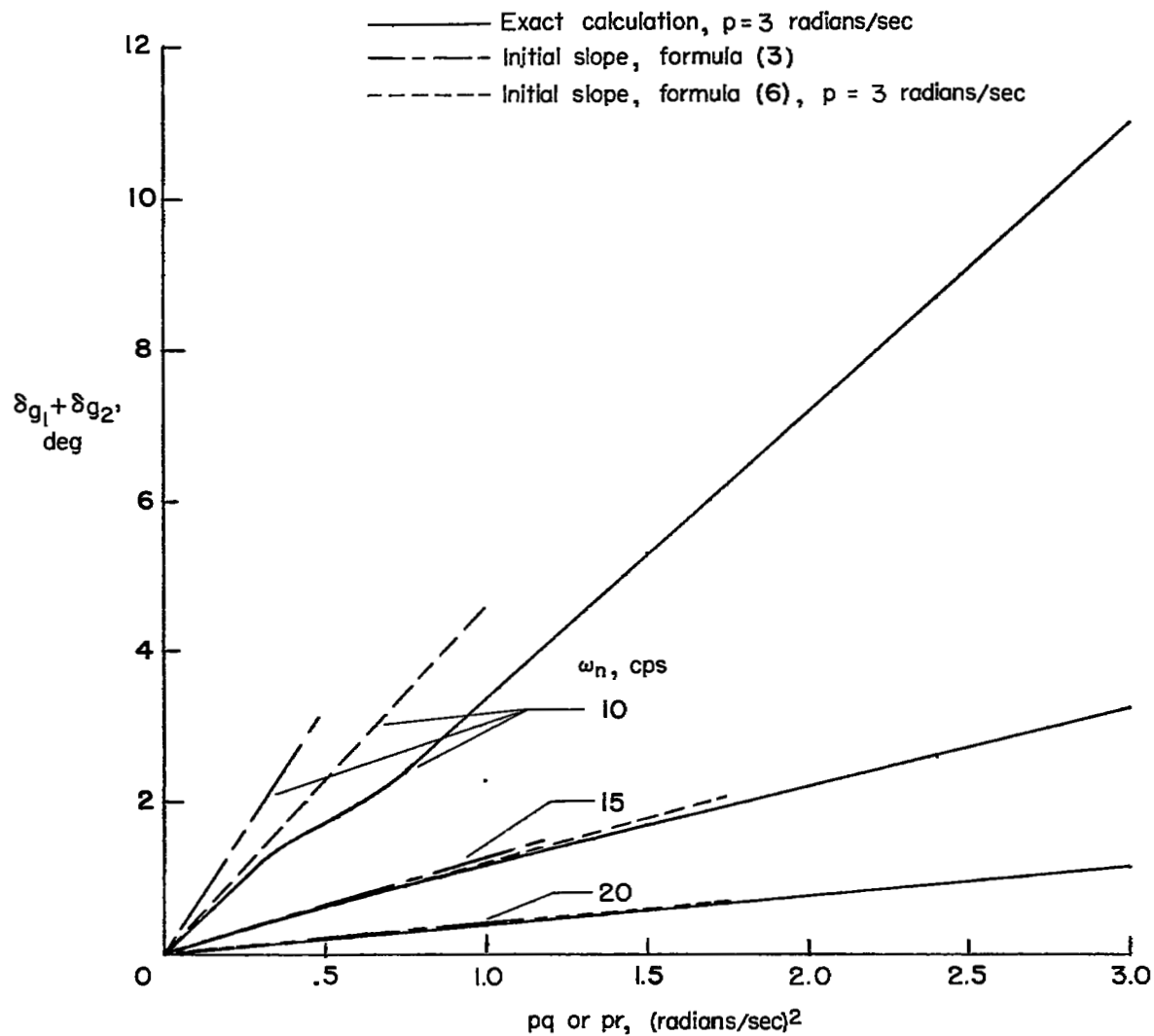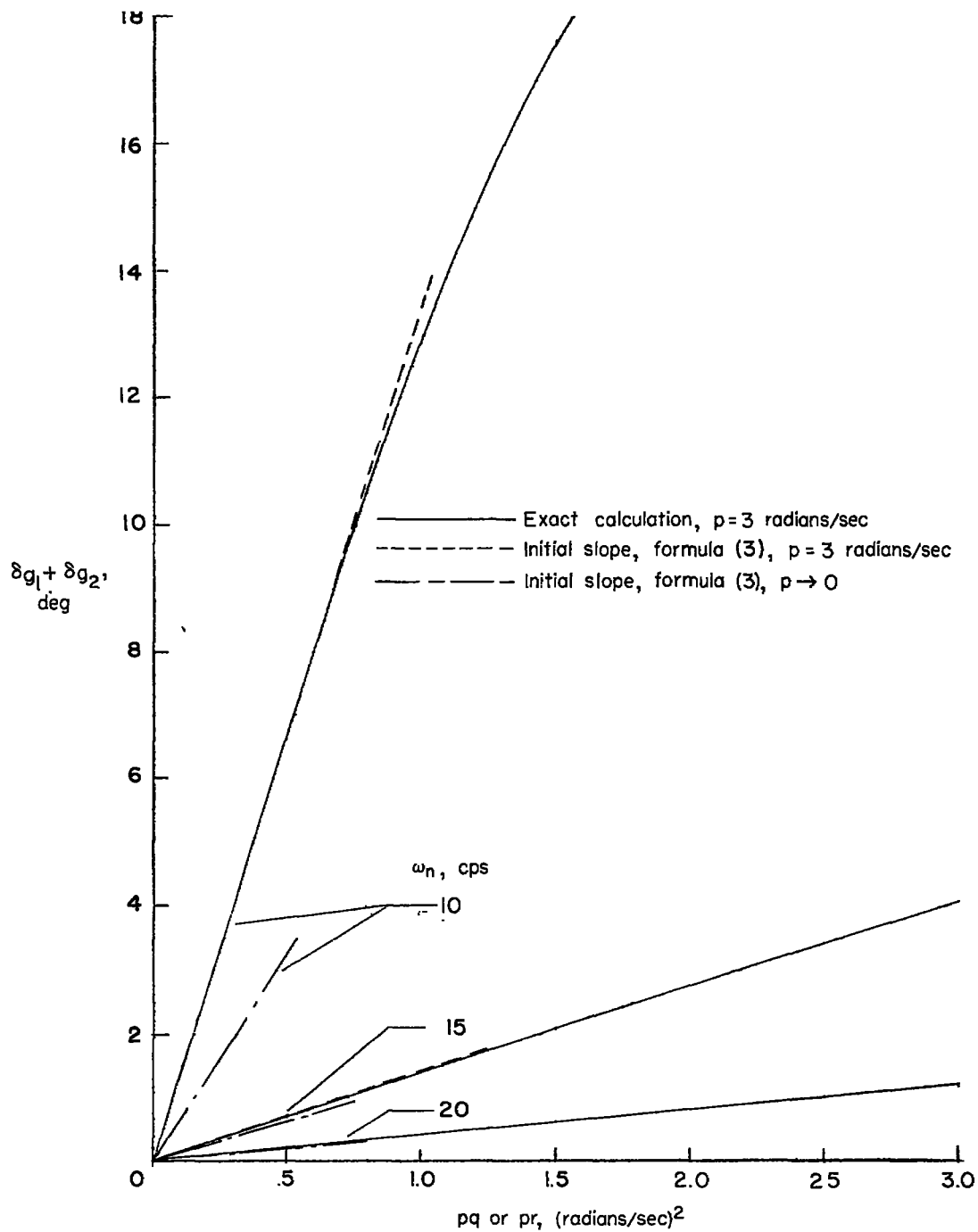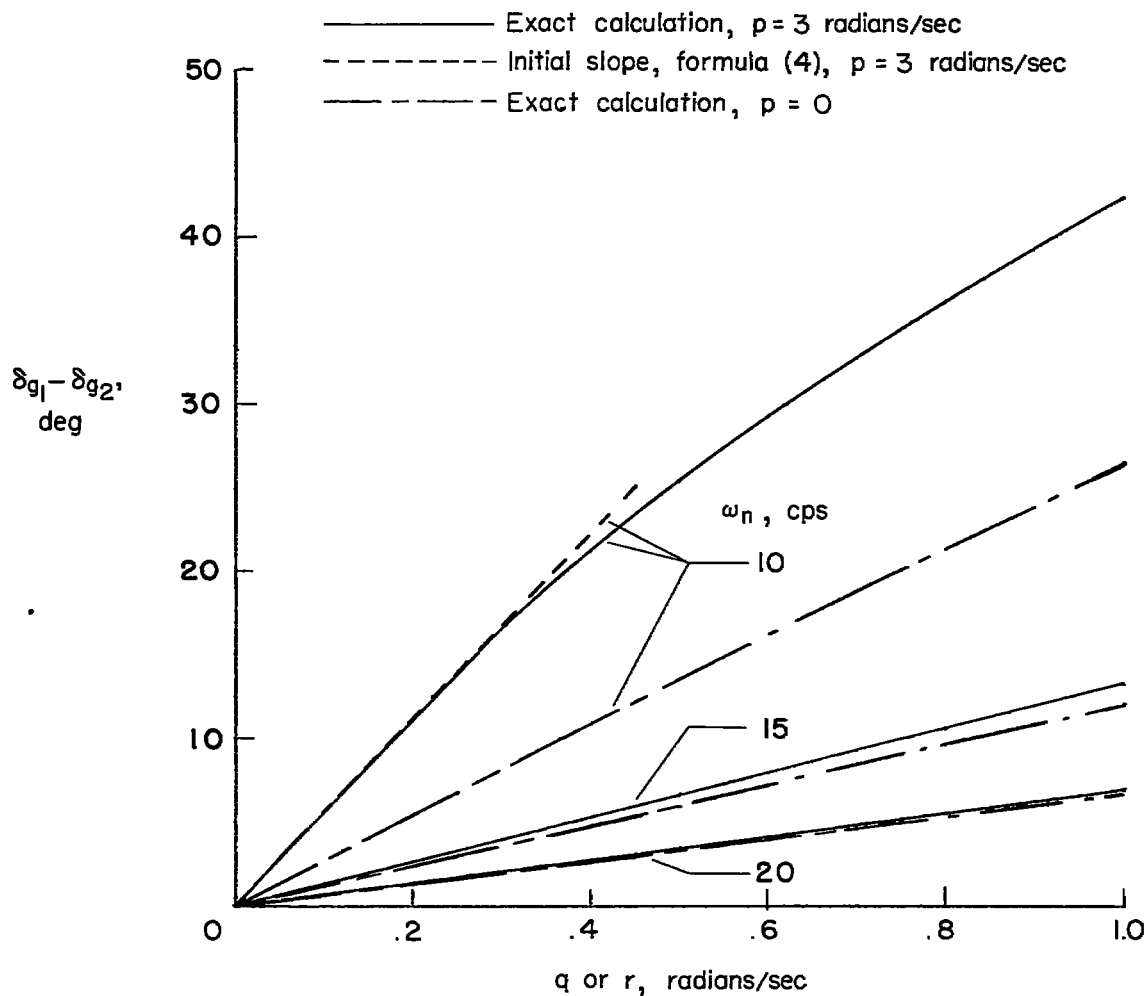ties $\omega_1\omega_2$. Gyros spin in opposite directions.

Figure 10.- Calibration curves for gyroscopic instrument for measuring pq and pr. Gyro oriented with spin axis parallel to pitch or yaw axis.

(a) Gimbal deflection $\delta_{g_1} + \delta_{g_2}$ as a function of pq or pr.

Figure 11.- Calibration curves for gyroscopic instrument for measuring pq and pr. Gyros oriented with spin axis parallel to roll axis.

(b) Gimbal deflection $\delta_{g_1} - \delta_{g_2}$ as a function of q or r.

Figure 11.- Concluded.